# Teaching Plan (DSC-7: Group Theory): B.Sc. (Hons.) Mathematics, Semester-3

Weeks 1 and 2: Permutation groups and group of symmetries, Cycle notation for permutations and properties, Even and odd permutations, Alternating groups. [1]: Chapter 5 (up to Theorem 5.7, page 104).

**Weeks 3 and 4:** Cosets and its properties, Lagrange's theorem and consequences including Fermat's Little theorem, Number of elements in product of two finite subgroups. [1]: Chapter 7 (up to Example 6, page 144).

**Weeks 5 and 6:** Normal subgroups, Factor groups, Cauchy's theorem for finite Abelian groups. [1]: Chapter 9 (Theorems 9.1, 9.2, 9.3 and 9.5, and Examples 1 to 12).

Weeks 7 and 8: Group homomorphisms, isomorphisms and properties, Cayley's theorem. [1]: Chapter 10 (Theorems 10.1 and 10.2, Examples 1 to 11). [1]: Chapter 6 (Theorems 6.1, 6.2, 6.3, and Examples 1 to 10).

Week 9: First, Second and Third isomorphism theorems for groups. [1]: Chapter 10 (Theorems 10.3, 10.4, Examples 12 to 15, and Exercises 41 and 42, page 208 for second and third isomorphism theorems for groups).

Weeks 10 and 11: Automorphism, Inner automorphism, Automorphism groups, Automorphism groups of cyclic groups, Applications of factor groups to automorphism groups.[1]: Chapter 6 (Page 128 to132).[1]: Chapter 9 (Theorem 9.4, and Example 16).

Weeks 12 and 13: External direct products of groups and its properties, The group of units modulo *n* as an external direct product, Applications to data security and electric circuits. [1]: Chapter 8.

Weeks 14 and 15: Internal direct products; Fundamental theorem of finite Abelian groups and its isomorphism classes.

[1]: Chapter 9 (Section on internal direct products, pages 183 to 187).

[1]: Chapter 11 (Outline of the proof of Fundamental theorem of finite Abelian groups, and its application to determine the isomorphism classes of Abelian groups).

#### **Essential Reading**

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.

## Teaching Plan (DSC-8: Riemann Integration): B.Sc. (Hons.) Mathematics, Semester-3

Weeks 1 and 2: Definition of upper and lower Darboux sums, Darboux integral, Inequalities for upper and lower Darboux sums.

[1]: Chapter 6 (Sections 32.1 to 32.4).

Weeks 3 to 5: Necessary and sufficient conditions for the Darboux integrability; Riemann's definition of integrability by Riemann sum and the equivalence of Riemann's and Darboux's definitions of integrability.

[1]: Chapter 6 (Sections 32.5 to 32.10).

Week 6: Definition and examples of the Riemann-Stieltjes integral. [1]: Chapter 6 (Sections 35.1, and 35.2).

Weeks 7 to 9: Riemann integrability of monotone functions and continuous functions, Properties of Riemann integrable functions.

[1]: Chapter 6 (Sections 33.1, and 33.6).

Week 10: Definitions of piecewise continuous and piecewise monotone functions and their Riemann integrability; Intermediate value theorem for integrals, [1]: Chapter 6 (Sections 33.7 to 33.9, and Exercise 33.14).

Week 11: Fundamental Theorems of Calculus (I and II). [1]: Chapter 6 (Sections 34.1 to 34.3).

Weeks 12 and 13: Methods of integration: integration by substitution and integration by parts; Volume by slicing and cylindrical shells, Length of a curve in the plane and the area of surfaces of revolution. [2]: Chapter 4 (Section 4.9), Chapter 7 (Section 7.2), and Chapter 5 (Sections 5.2 to 5.5).

Weeks 14 and 15: Improper integrals of Type-I, Type-II and mixed type, Convergence of improper integrals, The beta and gamma functions and their properties.

[3]: Chapter 7 (Section 7.8).

[4]: Chapter 9 [Sections 9.5 (up to examples 9.47, page 395), and 9.6 (pages 405 to 408).

- 1. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer.
- 2. Anton, Howard, Bivens Irl and Davis Stephens (2012). Calculus (10th ed.). John Wiley & Sons, Inc.
- 3. Denlinger, Charles G. (2011). Elements of Real Analysis, Jones & Bartlett India Pvt. Ltd.
- 4. Ghorpade, Sudhir R. and Limaye, B. V. (2006). A Course in Calculus and Real Analysis. Undergraduate Texts in Mathematics, Springer (SIE). Indian Reprint.

## Teaching Plan (DSC-9: Discrete Mathematics): B.Sc. (Hons.) Mathematics, Semester-3

Week 1: The cardinality of a set. [2] Chapter 3 (Section 3.3).

Weeks 2 and 3: Definitions, examples and basic properties of partially ordered sets, Order-isomorphisms, Covering relations, Hasse diagrams.

[1]: Chapter 1 (Sections 1.1 to 1.5, Section 1.6 (up to second bullet page 4), Sections 1.14 to 1.18). [3]: Chapter 1 (Subsection 1.1).

Weeks 4 and 5: Dual of an ordered set, Duality principle, Bottom and top elements, Maximal and minimal elements, Zorn's lemma, Building new ordered sets, Maps between ordered sets.

[1]: Chapter 1 (Sections 1.19 to 1.24, Section 1.25 (only definition of product of partially ordered sets and diagrams to be done), Sections 1.26, 1.34, 1.35(1), and 1.36).

[1]: Chapter 2 (Sections 2.1 to 2.2); [3]: Chapter 1 (Subsections 1.2 to 1.4).

Weeks 6 and 7: Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products, Lattice isomorphism.

[1]: Chapter 2 (Sections 2.3 to 2.5, 2.6 (excluding portion on down-set and up-set), 2.7 (only definition of lattices Sub *G* and *N*-Sub *G* to be done), 2.8 to 2.19, 2.22 to 2.25; all results to be stated without proof).
[3]: Chapter 1 (Subsections 1.5 to 1.20).

Weeks 8 and 9: Definitions, examples and properties of modular and distributive lattices. [1]: Chapter 4 (Sections (4.1 to 4.9); [3]: Chapter 1 (Subsections 2.1 to 2.6).

**Week 10:** The M<sub>3</sub>–N<sub>5</sub> theorem with applications, Complemented lattice, Relatively complemented lattice, Sectionally complemented lattice.

[1]: Chapter 4 (Section 4.10 (result to be stated without proof), and Section 4.11).

[3]: Chapter 1 (Subsections 2.7, 2.8 (except example(v)), 2.9 -2.14).(Results in 2.12, and 2.13 to be stated without proof)

Weeks 11 and 12: Boolean algebras, De Morgan's laws, Boolean homomorphism, Representation theorem, Boolean polynomials, Boolean polynomial functions, Equivalence of Boolean polynomials.

[3]: Chapter 1 [Subsections 3.1 to 3.8, and 3.9 (example(i); example (ii) and (iii) both without proofs); For 3.10 to 3.16 (Definitions and examples to be done. All results to be stated without proofs.)].

[3]: Chapter 1 [Subsections 4.1 to 4.10 (Definitions and examples to be done. All results to be stated without proofs)].

Weeks 13 and 14: Disjunctive normal form and conjunctive normal form of Boolean polynomials; Minimal forms of Boolean polynomials, Quine-McCluskey method, Karnaugh diagrams.

[3]: Chapter 1 [Subsections 4.11 to 4.14, 4.16 to 4.18 (Definitions and examples to be done. All results to be stated without proofs)]. [3]: Chapter 1 [Subsections 6.1 to 6.6 (Definitions and examples to be done. All results to be stated without proofs)].

**Week 15:** Switching circuits and applications, Applications of Boolean algebras to logic, set theory and probability theory. [3]: Chapter 2 [Subsections 7.1 to 7.5; 8.1, 8.3 to 8.5; 9.1 to 9.13, 9.14{(i) to (iii)}].

- 1. Davey, B. A., & Priestley, H. A. (2002). Introduction to Lattices and Order (2nd ed.). Cambridge University press, Cambridge.
- 2. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.
- 3. Lidl, Rudolf & Pilz, Gunter. (2004). Applied Abstract Algebra (2nd ed.), Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint.

## Teaching Plan (DSE-1(i): Graph Theory): B.Sc. (Hons.) Mathematics, Semester-3

Week 1: Definition, Examples and basic properties of graphs, Subgraphs, Pseudographs, Complete graphs, Bipartite graphs. [1]: Chapter 9 (Sections 9.1, and 9.2).

Weeks 2 and 3: Isomorphism of graphs, Paths and circuits, Connected graphs, Eulerian circuits, Hamiltonian cycles.

[1]: Chapter 9 (Section 9.3), Chapter 10 (Sections 10.1, and 10.2) (Theorems 10.1.4, 10.1.5, 10.2.4, and 10.2.6 without proof, exclude 10.2.3).
[2]: Chapter 1 (Theorem 1.12).

**Week 4:** Adjacency matrix, Weighted graph, Travelling salesman problem, Shortest path, Dijkstra's algorithm. [1] Chapter 10 (Sections 10.3, and 10.4) (Section 10.4 applications only)

Weeks 5 and 6: Applications of Path and Circuits: The Chinese Postman Problem, Digraphs, Bellman-Ford Algorithm. [1] Chapter 11 (Sections 11.1, and 11.2).

Week 7: Tournaments, Scheduling Problem. [1] Chapter 11 (Sections 11.4, and 11.5)

Week 8: Trees, Properties of Trees, Spanning Trees. [1] Chapter 12 (Sections 12.1, and 12.2) (Theorem 12.2.3 statement only). (do *Forest* from Exercise 12.1 question 26).

**Week 9:** Minimum spanning tree algorithms. [1] Chapter 12 (Section 12.3). (Kruskal's algorithm, Prim's algorithm, and bound for minimum Hamiltonian cycle).

Weeks 10 and 11: Cut-vertices, Blocks and their characterization, Connectivity and edgeconnectivity.

[2] Chapter 4 (Theorem 4.1), Chapter 5 (Section 5.1, 5.2 and Section 5.3 up to Theorem 5.11)

Week 12 and 13: Planar graphs, Euler's formula, Kuratowski theorem, Graph coloring and applications.

[1] Chapter 13 (Sections 13.1, and 13.2).

Weeks 14 and 15: Matchings, Hall's Theorem, Independent sets and covers. [2] Chapter 8 (Section 8.1, exclude Theorem 8.4).

- 1. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.
- 2. Chartrand, Gary, & Zhang, Ping (2012). A First Course in Graph Theory. Dover Publications.

### Teaching Plan (DSE-1(ii): Mathematical Python): B.Sc. (Hons.) Mathematics, Semester-3

Weeks 1 and 2: Review of Python fundamentals; Drawing diverse shapes using code and Turtle.[2]: Chapter 1 (Review: Fundamentals of Python).[1]: Chapters 1 to 3.

Weeks 3 and 4: Using matplotlib and NumPy for data organization, Structuring and plotting lines, bars, markers, contours and fields, managing subplots and axes; Pyplot and subplots.[3]: Chapter 2 (up to page 45).

- https://patrickwalls.github.io/mathematicalpython/scipy/numpy/
- https://patrickwalls.github.io/mathematicalpython/scipy/matplotlib/
- https://matplotlib.org/stable/gallery/lines bars and markers/index.html
- https://matplotlib.org/stable/gallery/images contours and fields/index.html
- https://matplotlib.org/stable/gallery/subplots axes and figures/index.html
- https://matplotlib.org/stable/tutorials/introductory/pyplot.html

Week 5: Animations of decay, Bayes, Random walk.

[3]: Chapter 5 (Generating random numbers, pages 134 to 139; page 136 is optional).

- https://matplotlib.org/stable/gallery/animation/index.html
- https://matplotlib.org/stable/gallery/animation/animate\_decay.html
- https://matplotlib.org/stable/gallery/animation/bayes update.html
- https://matplotlib.org/stable/gallery/animation/random walk.html

**Week 6:** NumPy for scalars and linear algebra on *n*-dimensional arrays; Computing eigenspace. [2]: Chapter 4 (pages 226 to 229).

- https://numpy.org/numpy-tutorials/content/tutorial-svd.html
- https://patrickwalls.github.io/mathematicalpython/linear-algebra/eigenvalues-eigenvectors/

**Week 7:** Solving dynamical systems on coupled ordinary differential equations, Functional programming update fundamentals using NumPy.

- https://patrickwalls.github.io/mathematicalpython/differential-equations/first-order/
- https://patrickwalls.github.io/mathematicalpython/differential-equations/systems/
- https://realpython.com/python-functional-programming/

Weeks 8 and 9: Symbolic computation and SymPy: Differentiation and integration of functions, Limits. [3]: Chapter 4 (up to page 96), and Chapter 7.

- https://docs.sympy.org/latest/guides/index.html
- https://docs.sympy.org/latest/tutorials/intro-tutorial/calculus.html

Week 10: Solution of ordinary differential equations, Computation of eigenvalues, Solution of expressions at multiple points (lambdify).

- https://docs.sympy.org/latest/guides/solving/solve-ode.html
- https://docs.sympy.org/latest/tutorials/intro-tutorial/matrices.html
- https://docs.sympy.org/latest/modules/utilities/lambdify.html

Week 11: Simplification of expressions, Factorization, Collecting and canceling terms, Partial fraction decomposition, Trigonometric simplification, Exponential and logarithms, Series expansion and finite differences, Solvers, Recursive equations.

- [3]: Chapter 4
- [2]: Chapter 5.
- [1]: Chapter 6, and 10.
- https://docs.sympy.org/latest/modules/solvers/solvers.html

Weeks 12 and 13: Pretty printing using SymPy; Pandas API for IO tools: interfacing Python with text/csv, HTML, LaTeX, XML, MSExcel, OpenDocument, and other such formats.

[3]: Chapter 4 (pages 97-100).

- https://docs.sympy.org/latest/tutorials/intro-tutorial/printing.html
- [2]: Chapter 2 (pages 73-83).
- https://pandas.pydata.org/docs/user\_guide/io.html

Week 14 and 15: PyLaTeX and writing document files from Python with auto-computed values, Plots and visualizations.

- https://pypi.org/project/PyLaTeX/
- https://matplotlib.org/stable/tutorials/text/usetex.html
- https://pandas.pydata.org/docs/user\_guide/visualization.html

#### **Essential Readings**

- 1. Farrell, Peter (2019). Math Adventures with Python. No Starch Press. ISBN Number: 978-1-59327-867-0.
- 2. Farrell, Peter et al. (2020). The Statistics and Calculus with Python Workshop. Packet Publishing Ltd. ISBN: 978-1-80020-976-3.
- 3. Saha, Amit (2015). Doing Math with Python. No Starch Press. ISBN: 978-1-59327-640-9

#### **Computer Lab work:**

Weeks 1 to 4:

- Spyder Environment preparation with download, installation of required components.
- Implementation of turtle draw for polygonal shapes.
- Using lists and loops for common functions.
- List manipulation.
- Animating objects.
- Interactive grid.
- Drawing complex patterns.

Week 5: Animated plots for solution of problems: decay function w. r. t. time, conditional probability and bayes rule, random walk.

Week 6: Solution of linear algebra problems: Systems of equations, eigenvalues and eigenvectors.

Week 7: Newton's law of cooling, Coupled ODEs with initial conditions.

Weeks 8 and 9: Examples of limits, differentiation and integration of functions.

Weeks 10 to 15: Examples from text references, and similar to those from online sources.

## Teaching Plan (DSE-1(iii): Number Theory): B.Sc. (Hons.) Mathematics, Semester-3

**Week 1:** The Euclidean Algorithm and linear Diophantine equation. [1]: Chapter 2 (Section 2.4 up to page 28, and Section 2.5).

Weeks 2 and 3: Least non-negative residues and complete set of residues modulo *n*; Linear congruences, The Chinese remainder theorem, and system of linear congruences in two variables. [1]: Chapter 4 (Section 4.2 page 64, and Section 4.4).

**Week 4:** Fermat's little theorem, Wilson's theorem and its converse, Application to solve quadratic congruence equation modulo odd prime *p*.

[1]: Chapter 5 (Section 5.2 up to before pseudo-prime at page 90, and Section 5.3).

Weeks 5 and 6: Number-theoretic functions for the sum and number of divisors, Multiplicative function, Möbius inversion formula and its properties; Greatest integer function with an application to the calendar.

[1]: Chapter 6 (Sections 6.1, 6.2, 6.3 up to page 118, and 6.4).

**Weeks 7 and 8:** Euler's Phi-function, Euler's theorem and some properties of the Phi-function. [1]: Chapter 7 (Section 7.2, Theorem 7.2 without proof, Section 7.3, and Section 7.4, Theorem 7.6 without proof).

Weeks 9 to 11: The order of an integer modulo n and primitive roots for primes, Primitive roots of composite numbers n: when n is of the form  $2^k$ , and when n is a product of two coprime numbers. [1]: Chapter 8 (Sections 8.1, 8.2, and 8.3 (up to page 159)).

Week 12: The quadratic residue and nonresidue of an odd prime and Euler's criterion. [1]: Chapter 9 (Section 9.1).

Weeks 13 and 14: The Legendre symbol and its properties, Quadratic Reciprocity law and its application.

[1]: Chapter 9 (Section 9.2 up to page 181, Statement of Theorems 9.3 and 9.5, and Section 9.3).

Week 15. Introduction to cryptography, Hill's cipher, Public-key cryptography and RSA encryption and decryption technique.

[1]: Chapter 10 (Section 10.1).

## **Essential Reading**

1. Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint 2017.

# Teaching Plan (DSC-3, and DSE-1(iii): Theory of Equations and Symmetries):

B.A. (Prog.) with Mathematics as Major, and B.Sc. (Physical Sc./Mathematical Sc.), Sem-3.

Weeks 1 and 2: General properties of polynomials and equations; Statement of the Fundamental theorem of algebra and its consequences.

- [1] Chapter I (Sections 8, 9 and 10); Chapter II (Sections 12 to 17).
- [2] Chapter II (Sections 13 to 19)

Weeks 3 and 4: Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots.

- [1] Chapter II (Sections 18 to 22).
- [2] Chapter II (Sections 21, 24, 25 and 27), and Chapter VI [Section 67] (Proofs of theorems in the Chapters II and VI are omitted).

Weeks 5 and 6: Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given.

[1] Chapter III (Sections 23 and 24).

[2] Chapter II (Sections 20).

**Weeks 7 and 8:** De Moivre's theorem for rational indices, the *n*th roots of unity and symmetries of the solutions; Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms.

[2] Chapter I (Sections 7 to 10).

[1] Chapter III (Section 26); Chapter IV (Sections 29 to 34).

Weeks 9 and 10: Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.

[1] Chapter VI (Sections 56 and 64).

[2] Chapter IV (Sections 42, 43, 51 and 52).

Weeks 11 and 12: Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots.

[2] Chapter IX (Sections 103 to 106, methods only).

[1] Chapter VIII (Section 77, method only).

Weeks 13 to 15: Computation of symmetric functions such as:

$$\sum \alpha^2 \beta$$
,  $\sum \alpha^2 \beta^2$ ,  $\sum \alpha^2 \beta \gamma$ ,  $\sum \frac{1}{\alpha^2 \beta \gamma}$ ,  $\sum \alpha^{-3}$ ,  $\sum (\beta + \gamma - \alpha)^2$ ,  $\sum \frac{\alpha^2 + \beta \gamma}{\beta + \gamma}$ ,... of polynomial equations;

Transformation of equations by symmetric functions and in general.

[1] Chapter III (Sections 27 and 28); Chapter IV (Sections 39, 41 and 44).

[2] Chapter IX (Section 109, methods only).

- 1. Burnside, W.S., & Panton, A.W. (1979). *The Theory of Equations* (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
- 2. Dickson, Leonard Eugene (2009). *First Course in the Theory of Equations*. John Wiley & Sons, Inc. The Project Gutenberg eBook: http://www.gutenberg.org/ebooks/29785

#### Teaching Plan (Discipline A-3, and GE-3(i): Differential Equations):

B.A. (Prog.) with Mathematics as Major, B.A./B.Sc. (Prog.) with Mathematics as non-Major, B.Sc. (Physical Sciences/Mathematical Sciences), and Generic Electives- Semester-3.

Weeks 1 and 2: First order ordinary differential equations: Basic concepts and ideas, First order exact differential equations, Integrating factors and rules to find integrating factors.[2]: Chapter 1 (Sections 1.1, and 1.2), Chapter 2 (Sections 2.1, 2.2, and 2.4 up to page 64).

Week 3: Linear equations and Bernoulli equations, Initial Value Problems, Applications of first order differential equations: Orthogonal trajectories and Rate Problems.[2]: Chapter 2 (Sections 2.3), Chapter 3 (Section 3.1 up to page 74, and Section 3.3 up to page 94).

Weeks 4 and 5: Basic theory of higher order linear differential equations, Wronskian and its properties.

[2]: Chapter 4 (Sections 4.1 up to page 115).

Weeks 6 and 7: Linear homogeneous equations with constant coefficients, Linear non-homogeneous equations, Method of undetermined coefficients. [2]: Chapter 4 (Section 4.1 from page 120 onwards, Sections 4.2, and 4.3).

Weeks 8 and 9: Method of variation of parameters (only second order), Two-point boundary value problems, Cauchy- Euler equations, Systems of linear differential equations.

[2]: Chapter 4 (Sections 4.4, and 4.5).

[2]: Chapter 1 (Section 1.3 up to page 16).

[2]: Chapter 7 (Sections 7.1, and 7.3).

Weeks 10 and 11: Partial differential equations: Basic concepts and definitions, Classification and construction of first-order partial differential equations, Method of characteristics and general solutions of first order partial differential equations.

[1]: Chapter 2 (Sections 2.1 to 2.3, and 2.5).

Weeks 12 and 13: Canonical forms and method of separation of variables for first-order partial differential equations.

[1]: Chapter 2 (Sections 2.6, and 2.7).

Weeks 14 and 15: Classification and reduction to canonical forms of second-order linear partial differential equations and their general solutions.

[1]: Chapter 4 (Sections 4.1 to 4.4).

- 1. Myint-U, Tyn and Debnath, Lokenath (2007). Linear Partial Differential Equations for Scientist and Engineers (4th ed.). Birkhäuser. Indian Reprint.
- 2. Ross, Shepley L. (1984). Differential Equations (3rd ed.). John Wiley & Sons.

Teaching Plan (DSE-1(i): Combinatorics): B.Sc. (Physical Sc./Mathematical Sc.), Sem-3.

Week 1: Basic counting principles. [2]: Chapter 5 (Section 5.1).

Weeks 2 and 3: Permutations and Combinations (with and without repetitions), Binomial coefficients, Counting subsets of size k.

[2]: Chapter 5 [Sections 5.2 (up to Example 5), 5.3 (up to Theorem 2), 5.4 (Example 1, and Example 2), and 5.5 (up to Example 1)].

Weeks 4 and 5: Multinomial coefficients, Set-partitions, The inclusion-exclusion principle and applications.

[1]: Chapter 1 [Section 1.3 (Only Definition), Theorem 1.3.5].

[1]: Chapter 4 (Section 4.1).

Weeks 6 and 7: Generating functions: Generating function models, Calculating coefficients of generating functions, Polynomial expansions, Binomial identity. [2]: Chapter 6 (Sections 6.1, and 6.2).

Weeks 8 and 9: Exponential generating functions, Recurrence relations: Recurrence relation models.

[2]: Chapter 6 (Section 6.4).

[2]: Chapter 7 [Section 7.1 (up to Example 5)].

Weeks 10 and 11: Divide-and-conquer relations, Solution of linear recurrence relations, Solutions by generating functions. [2]: [Chapter 7 (Sections 7.2, 7.3, and 7.5).

Weeks 12 to 14: Partition theory of integers: Ordered partition, Unordered partition, Ferrers diagram, Conjugate of partition, Self-conjugate partition. [1]: Chapter 13 (Sections 13.1.1 to 13.1.9).

Week 15: Durfee square, Euler's pentagonal theorem. [1]: Chapter 13 (Definition 13.2.1, and Theorem 13.3.1).

- 1. Sane, Sharad S. (2013). Combinatorial Techniques. Hindustan Book Agency (India).
- 2. Tucker, Alan (2012). Applied Combinatorics (6th ed.). John Wiley & Sons, Inc.

#### Teaching Plan (DSE-1(ii): Elements of Number Theory): B.Sc. (Physical Sc./Math Sc.), Sem-3.

Weeks 1 and 2: Revisiting: The division algorithm, divisibility and the greatest common divisor.Euclid's lemma; The Euclidean algorithm, Linear Diophantine equations.[1]: Chapter 2 (Sections 2.2, 2.3, 2.4 (up to page 28), and 2.5. All Theorems without proofs).

Weeks 3 and 4: The Fundamental theorem of arithmetic, The sieve of Eratosthenes, Euclid's theorem and the Goldbach conjecture; The Fibonacci sequence and its nature.

[1]: Chapter 3 (Sections 3.1 (Theorem 3.2 without proof), 3.2 (Theorem 3.4), and 3.3 (up to p 53)). [1]: Chapter 14 (Sections 14.1, and 14.2 (All results without proofs)).

Week 5: Congruence relation and its basic properties. [1]: Chapter 4 (Section 4.2).

Weeks 6 and 7: Linear congruences and the Chinese remainder theorem, System of linear congruences in two variables.

[1]: Chapter 4 (Section 4.4, Theorems 4.8 and 4.9 without proofs).

**Weeks 8 and 9:** Fermat's little theorem and its generalization, Wilson's theorem and its converse. [1]: Chapter 5 (Section 5.2 up to before pseudo-prime at Page 90, Section 5.3 before Theorem 5.5).

**Week 10:** Number-theoretic functions for sum and the number of divisors of a positive integer, Multiplicative functions. [1]: Chapter 6 (Section 6.1, All Theorems without proofs).

[1]: Chapter 6 (Section 6.1, All Theorems without proofs).

Week 11: The greatest integer function; Euler's phi-function and its properties.[1]: Chapter 6 (Section 6.3 up to page 118)[1]: Chapter 7 (Section 7.2, Theorem 7.2 without proof).

Weeks 12 and 13: Basics of cryptography, Hill's cipher, Public-key cryptosystems and RSA encryption and decryption technique. [1]: Section 10.1.

Weeks 14 and 15: Introduction to perfect numbers, Mersenne numbers and Fermat numbers. [1]: Sections 11.2 (up to page 223), 11.3 (before Theorem 11.4), and 11.4 (before Theorem 11.10)).

#### **Essential Reading**

1. Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint 2017.

#### Teaching Plan (GE-3(ii): Lattices and Number Theory): Generic Elective, Semester-3.

Weeks 1 and 2: Definitions, Examples and basic properties of partially ordered sets, Order isomorphism, Hasse Diagram, Maximal and minimal elements, Dual of an ordered set, Duality principle; Statements of Well Ordering Principle and Zorn's Lemma.

[1]: Chapter 1 (Sections 1.1 to 1.5, 1.14, 1.15, 1.19, 1.20, 1.21, and 1.23).

[2]: Chapter 1 (Subsections 1.1 to 1.4).

Weeks 3 to 5: Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms, Distributive lattices.

[1]: Chapter 2 (Sections 2.1 to 2.5, 2.6 (excluding portion on down-set and up-set), 2.8 to 2.19, all results to be stated without proofs).

[2]: Chapter 1 (Subsections 1.5 to 1.18, and 2.1 to 2.6).

Weeks 6 and 7: Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, Switching circuits and applications.

[2]: Chapter 1 [Subsections 3.1 to 3.6, 4.1 to 4.3, and 6.1 to 6.6]. (Definitions and examples to be done. All results to be stated without proofs).

Weeks 8 and 9: The division algorithm: GCD, The Euclidean algorithm, Diophantine equation ax + by = c. Primes: The Fundamental Theorem of Arithmetic, Infinitude of primes, Twin primes and Goldbach conjecture. [3]: Chapter 2 (Sections 2.2, 2.3, 2.4 (up to page 28), and 2.5). Theorems 2.1 to 2.9 without proofs. [3]: Chapter 3 (Sections 3.1 (Theorem 3.2 without proof), 3.2 (Theorem 3.4), and 3.3 up to page 53).

Weeks 10 and 11: The theory of congruences: Basic properties and applications, Linear congruences and the Chinese Remainder Theorem, Fermat's Little Theorem and Wilson's Theorem.

- [3]: Chapter 4 (Sections 4.2, and 4.4 (Theorems 4.7 to 4.9 without proofs)).
- [3]: Chapter 5 (Section 5.2 up to before pseudo-prime at Page 90, Section 5.3 before Theorem 5.5). (All Theorems without proofs).

Weeks 12 and 13: Number-Theoretic Functions: Sum and number of divisors, Euler's Phi-function and Euler's generalization of Fermat's Little Theorem.

[3]: Chapter 6 (Section 6.1; All Theorems without proofs).

[3]: Chapter 7 (Sections 7.2, and 7.3; All Theorems without proofs).

Weeks 14 and 15: Primitive roots: The order of an integer modulo *n*, and primitive roots of an integer. Quadratic Reciprocity Law: Quadratic residue and nonresidue, Euler's Criterion, The Legendre symbol and its properties and Quadratic Reciprocity Law.

[3]: Chapter 8 (Sections 8.1, and 8.2 (Theorems and Corollaries without proofs).

[3]: Chapter 9 (Sections 9.1, 9.2 (up to page 181), and 9.3; All Theorems and Corollaries without proofs).

- 1. Davey, B A., & Priestley, H. A. (2002). Introduction to Lattices and Order (2nd ed.), Cambridge University Press, Cambridge.
- 2. Lidl, Rudolf & Pilz, Günter. (1998). Applied Abstract Algebra (2nd ed.), Undergraduate Texts in Mathematics, Springer (SIE), Indian Reprint 2004.
- 3. Burton, David M. (2012). Elementary Number Theory (7th ed.), Mc-Graw Hill Education Pvt. Ltd. Indian Reprint.